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The formation of a dip on the surface of a liquid draining from a tank

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Experiments were performed studying the formation of a dip on the surface of an initially stationary liquid draining from a cylindrical tank through an axisymmetrically placed circular orifice. Based upon the information obtained from the experiments, a simple analytical expression was derived predicting the height of the liquid surface in the tank at which this dip forms. A comparison was made between the experimental data and the results of the analysis and good agreement was found between theory and data.

Introduction

Although the formation of a dip on the free surface of a liquid draining from a container is often observed, relatively little is known about the details of this phenomenon. The theoretical analyses of Dergarabedian (1960), Miles (1962), Saad & Oliver (1964) and Bhuta & Koval (1965) indicate that the shape of the

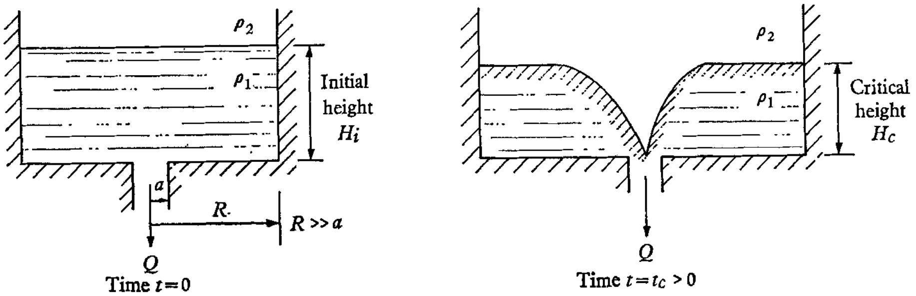

FIGURE 1. The formation of a dip on the surface of a draining liquid.

free surface is strongly affected by the drain. The results of Saad & Oliver and Bhuta & Koval show that draining introduces oscillations in an initially stationary free surface, while Miles found that free-surface oscillations initially present are damped by the drainage of the liquid. These analyses do not describe explicitly the conditions at which the dip on the liquid surface reaches the level of the bottom of the container and the drain (see figure 1).

In this investigation experiments were performed to study the parameters influencing the formation of a dip on the free surface of an initially stationary liquid draining from a cylindrical tank. Based upon the experimental information obtained, a simple analysis is presented indicating the criteria necessary for the dip to extend into the drain.

Experimental

When a liquid drains from a circular tank through an axisymmetrically placed circular orifice (drain) a 'dip' develops on the free surface of the liquid as the surface level reaches a certain height (H_c in figure 1). Here the formation of this dip was studied with the apparatus described below.

Two Plexiglas cylinders of $5\frac{1}{2}$ and 9 in. inside diameters and 12 in. height with provisions for interchangeable drain holes served as the tank (figure 2). With tank A $\frac{1}{2}$ in. and $\frac{1}{4}$ in. diameter drains were used while in tank B the drain diameters employed were $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1 in. These diameters were selected in order to ensure that the radius of the tank, R, was large compared to the radius of the orifice, a, ($R/a \gg 1$) throughout the experiments.

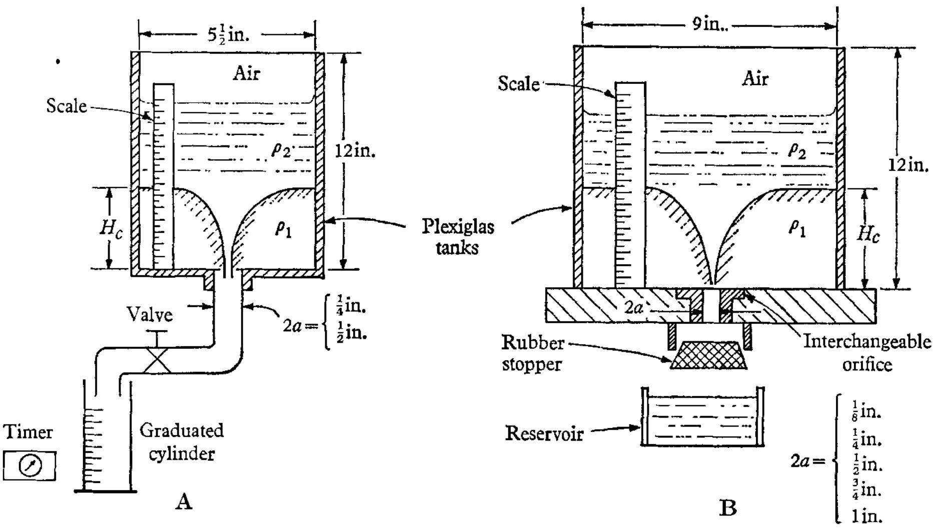

FIGURE 2. Schematic diagram of experimental apparatus.

In tank A the flow was controlled by a valve connected to the drain and the volume flow rate, Q, was measured with a graduated cylinder and timer. In tank B the fluid was allowed to flow freely through the drain into the reservoir and the instantaneous volume flow rate was calculated by measuring the height of the liquid in the tank and by assuming that the flow is everywhere inviscid. The height, H, of the free surface was measured visually at the outer radius of each tank with a scale. The liquid used in the experiments was water. However, in order to study the effects of surface tension and the effects of the respective viscosities and densities of the water and the fluid above it, experiments were performed with not only water in contact with air but also with fluids placed on top of the water (figures 2, 4). In the present experiments the depth of the top fluid (ΔH, figure 4) was kept constant at 1, $1\frac{1}{2}$ or 2 in. The fluids used are listed in table 1.

Prior to each experimental run the liquids were kept in the tank for several hours to ensure that there was no liquid motion when the drain valve was opened. As expected, the slightest disturbance in the liquid affected very strongly the height at which the dip formed.

Fluid 1 bottom	Fluid 2 top	Density (g/c.c.) ρ_1	Density (g/c.c.) ρ_2	Viscosity (cP) μ_1	Viscosity (cP) μ_2
Water	Air	~1·0	~0	~1·0	0·01
Water	Turpentine	1·0	0·87	1·0	1·30
Water	Silicone-oil	1·0	0·92	1·0	8·0
Water	Corn oil	1·0	0·92	1·0	60·0
Water	Kerosene	1·0	0·785	1·0	1·85

TABLE 1. Properties of fluids used in the experiments

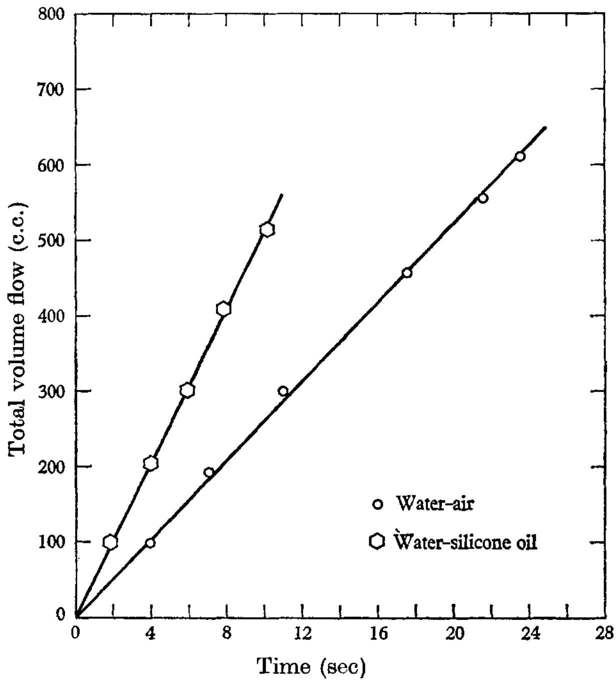

FIGURE 3. Volume flow through $\frac{1}{4}$ in. diameter drain in tank A. Valve fully open.

The following observations were made with the experimental system just described. For each of the liquid-liquid combinations tested (see table 1) the surface of the bottom fluid (water) appears to be smooth to the visual observer until a certain height is reached. At this height, referred to herein as critical height H_c (figures 1, 4) a dip forms above the drain on the surface of the bottom fluid and then grows so rapidly that it appears to the naked eye that the dip extends into the drain instantaneously. Furthermore, it has been observed that for any given liquid-liquid combination the critical height is independent of the initial height, H_i, of the liquid in the tank. Sample data demonstrating this effect are given in table 2. It has been observed also that the volume flow rate, Q, is nearly constant

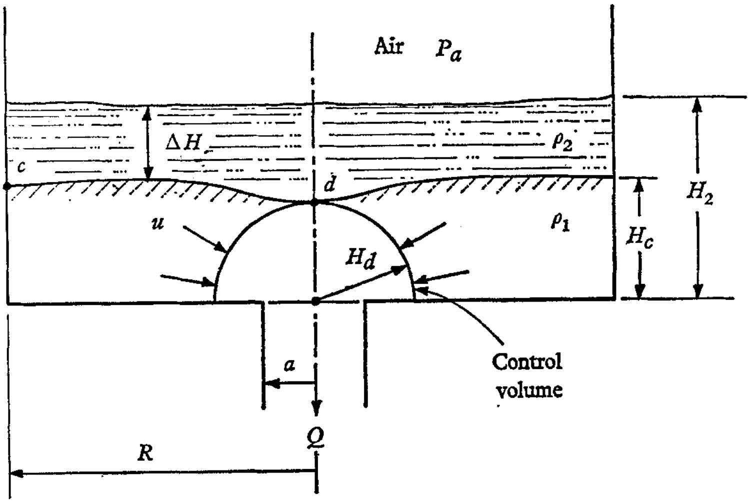

FIGURE 4. Definitions of symbols and control volume used in the analysis.

Drain radius a (in.)	Initial height H_i (in.)	Critical height H_c (in.)
0·5	2	0·82
0·5	10	0·82
0·375	2	0·42
0·375	10	0·40

TABLE 2. Sample data illustrating the effect of initial height of the water surface level on the critical height. Experimental data obtained with water open to air

up to the instant of dip formation (figure 3). This is to be expected for the large radius ratios, R/a, used in the present experiments (Sabersky & Acosta 1964). The latter two observations, namely that the critical height is independent of the initial height and that the volume flow rate is constant, suggest that an analysis based on the assumption of quasi steady flow may be successful in predicting the critical height.

Critical height

Based on the experimental observation described above an attempt will be made now to predict analytically the critical height at which the surface dip forms. The analysis is based on the following assumptions: (*a*) the effects of viscosities of both bottom (water) and top liquids are negligible; (*b*) the effect of surface tension at the interface of the top and bottom liquids is negligible; (*c*) the pressure at this interface is due to hydrostatic pressure only, i.e.

$$p_{1\mathrm{f}} = \rho_2 g \Delta H + p_a, \tag{1}$$

where p_{If} is the pressure at any point on the interface, g is the gravitational constant and all other symbols are identified in figure 4; (d) at the instant prior to the formation of the dip the flow is steady; (e) the initial depression (dip) on the surface develops so rapidly as to reach into the drain instantaneously. With the

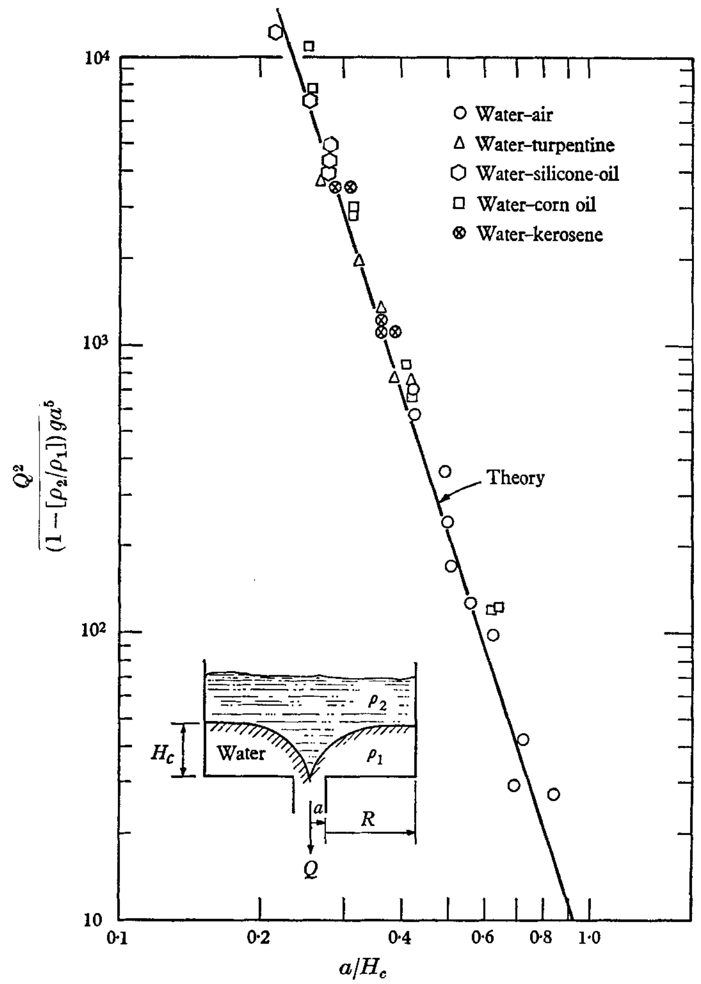

FIGURE 5. Critical height. Comparison between experimental data and theory, equation (6).

above assumptions at the instant of the formation of the initial depression the conservation of mass for the hemispherical control volume shown in figure 4 may be expressed as

$$Q = 2\pi U H_d^2, \tag{2}$$

and the Bernoulli equation along a streamline just below the interface is

$$p_c + \rho_1 g H_c = p_d + \rho_1 g H_d + \tfrac{1}{2}\rho_1 U^2, \tag{3}$$

where $U_c = 0$ and points c and d are shown in figure 4. In writing (2) and (3) it was further assumed that at every point on the surface of the control volume the flow velocity has the same value and is normal to this surface. Equations (1)–(3) may be rearranged to yield

$$H_c = H_d + \frac{Q^2}{8\pi^2(1-\rho_2/\rho_1)\,gH_d^4}. \tag{4}$$

In order to eliminate the height of the dip, H_d, from the above expression we shall utilize the experimental observation (assumption *e* above) that the dip grows very rapidly. Thus, we shall write that at the instant of dip formation

$$\frac{dH_c}{dt}\bigg/\frac{dH_d}{dt} \simeq 0. \tag{5}$$

Equations (4) and (5) give

$$\frac{H_c}{a} = 0{\cdot}69\left[\frac{Q^2}{(1-\rho_2/\rho_1)\,ga^5}\right]^{\frac{1}{5}}. \tag{6}$$

The radius of the orifice, a, has been arbitrarily selected as the length in normalizing (6). It is interesting to note that H_c, as given by (4), has a minimum value below which no solution exists for H_d from this equation. This minimum value is the same as the critical value of H_c derived above (6).

In order to test the validity of the assumptions employed in the derivation of the above expression, critical heights measured experimentally were compared to those calculated from (6). The results presented in figure 5 show good agreement between the data and the analytical results over a very wide range of the parameter $Q^2/(1-\rho_2/\rho_1)\,ga^5$. Thus, it appears that in systems similar to the ones used in this study the assumptions listed previously are reasonable and that (6) may be used to estimate the critical height at which the dip forms on the surface.

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REFERENCES

Bhuta, P. G. & Koval, L. R. 1965 *Sloshing of a liquid in a Draining or Filling Tank Under Variable g Conditions.* Symp. on Fluid Mech. and Heat Transfer under Low Gravitational Conditions, Lockheed Res. Lab. Palo Alto, California.

Dergarabedian, P. 1960 *The Behavior of Vortex Motion in an Emptying Container,* pp. 47–61. Proc. Heat Transfer and Fluid Mech. Inst., Stanford Univ. Press, Stanford, California.

Miles, J. W. 1962 Note on the damping of free-surface oscillations due to drainage. *J. Fluid Mech.* **12**, 438–40.

Saad, M. A. & Oliver, D. A. 1964 *Linearized Time-Dependent Free-Surface Flow in Rectangular and Cylindrical Tanks,* pp. 81–99. Proc. Heat Transfer and Fluid Mech. Inst., Stanford Univ. Press, Stanford, California.

Sabersky, R. H. & Acosta, A. J. 1964 *Fluid Flow,* p. 65. New York: MacMillan.